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Transition Regime Aerodynamic Heating of Missiles

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The three-dimensional flows over the ARIES missile at approximately 138-, 110-, and 91-km altitudes are simulated at zero angle of attack using the direct simulation Monte Carlo method. At these altitudes, the aerodynamics near the missile range from the transition to near free-molecule flow regimes. The aerodynamic heating predictions along the missile surface from the simulations are used to benchmark three empirical bridging relations. Furthermore, the effect of using an approximate free-molecular expression in the bridging relations is investigated. The values predicted using the approximate free-molecular expressions fail to predict any heat transfer along the cylindrical portions of the missile, and differ by as much as 90% along other sections. The bridging relations are found to predict heating rates within a factor of 2 of the simulated results, provided the full free-molecular expression from kinetic theory is used. It is found that no one bridging relation outperformed the others. Each has regions along the length of the missile, where it is in good agreement with the simulations, and other regions, where significant differences are observed.

Nomenclature

a	= speed of sound, m/s
a_c	= thermal accommodation coefficient
C_h	= local heat transfer coefficient, $2Q/\rho_\infty V_\infty^3$
H	= total enthalpy, J/kg
Kn	= Knudsen number
M	= Mach number
N	= number density, particles/m^3
Q	= convective heat transfer, W/m^2
R	= gas constant, J/kg K
Re	= Reynolds number
R_N	= nose radius, m
s	= speed ratio
T	= temperature, K
V	= total velocity magnitude, m/s
X	= mole fraction
x, y, z	= coordinate directions
α	= angle of attack, deg
γ	= ratio of specific heats
ε	= specular reflection coefficient
θ	= surface inclination angle, deg
μ	= absolute viscosity, kg/m s
ρ	= mass density, kg/m^3

Subscripts

br	= bridged values
c	= continuum values
FM	= free-molecular values
w	= surface values
∞	= freestream values

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Introduction

PREDICTING infrared emissions from a ballistic missile requires an accurate calculation of the skin temperature distribution over the entire missile surface. Accurate prediction of the missile skin temperature requires the determination of aerodynamic heating for a wide range of altitudes, velocities, and angles of attack. Arguably one of the least understood of the altitude ranges, for these types of problems is the transition regime, where typical continuum assumptions break down, yet free-molecular approximations are still invalid. Along its trajectory, a ballistic missile will experience different flow regimes, namely continuum, transition, and free molecule. The methods used to predict the aerodynamic heating differ in each of these regions because of the different physics involved. At lower altitudes early in the ascent portion or late in the descent portion of the trajectory, the missile is in the continuum regime where conventional computational fluid dynamics (CFD) solutions of the Navier–Stokes equations with appropriate boundary conditions are well established.[1] Furthermore, several approximate techniques are available, depending on the flow conditions, that allow for accurate and fast predictions of the aerodynamic heating.[1–3] In the middle portion of the ballistic trajectory, the missile experiences the free-molecular flow regime, where the effects of intermolecular collisions among the different molecules are negligible. In this region, kinetic theory offers closed-form analytic expressions for aerodynamic heating[4] that allow for fast and accurate heating predictions. Also, more approximate expressions (or limiting forms) are widely used that depend on the flow conditions and incidence angles to determine their validity.[5–7]

In the transition regime, the flow cannot be considered a continuum, and yet intermolecular collisions are important enough that free-molecular assumptions are invalid. To predict the transition regime aerodynamic heating, empirical bridging relations are usually employed.[6,7] These bridging relations attempt to approximate the aerodynamic heating by interpolating between the heating rates predicted using continuum and free-molecular approaches. However, these approximations are generally derived for and apply to blunt bodies and stagnation points, and are not recommended for use with pointed and slender bodies.[6,7] An initial comparison[8] showed large differences in transition regime aerodynamic heating predictions over ballistic missiles between bridging-relation approaches and a more accurate particle approach. In this paper we make a more consistent and improved application of bridging techniques

before making comparisons. Therefore, the present error estimates supersede those of Ref. 8.

The direct simulation Monte Carlo (DSMC) method of Bird[9] has proven to be an effective tool in calculating transitional flows with wide ranges of physical models, geometries, and boundary conditions.[10–15] Thus, DSMC simulations can provide excellent benchmarks to validate the bridging relations currently being employed. Then, a better understanding of when and where the current bridging relations will fail may be obtained. Furthermore, with the advances in computers and DSMC algorithms,[16,17] the method is now capable of providing the transitional data necessary for trajectory analyses.

In this paper, the ARIES missile is used as a working example. Three trajectory points corresponding to approximately 138-, 110-, and 91-km altitudes are examined. At these points, the ARIES missile was traveling at 2050, 2180, and 2250 m/s, respectively. At 138 km, the flow is quasi-free-molecular and the effects of using an approximate free-molecular formula are evaluated by comparing DSMC results with free-molecular calculations predicted by the full kinetic theory expressions. The differences between the free-molecular calculations are also discussed. Next, the DSMC simulation of the transitional flow over the missile for the 110-km altitude case was compared with predictions using three different bridging relations to assess their performance. The bridging relations used were from Matting,[6] from Nomura,[7] and a modified version of the bridging relation from Nomura suggested by Murty.[18] Differences between the bridging-relation predictions and the DSMC simulation are discussed. Lastly, because the 110-km case was seen to exhibit characteristics of a more rarefied transitional flow, the flow over the ARIES missile was simulated at 91 km to examine the performance of the bridging relations much deeper in the transition flow regime. The differences between the DSMC simulation and the different bridging relations are presented and discussed.

DSMC Method

The DSMC method used in this study was developed by Bird[9] and further modified by Rault[11,14,16,19] to allow for efficient simulations of complex three-dimensional geometries. Efficiency is obtained, in part, through the use of an unstructured grid overlaid on a Cartesian mesh. Using a Cartesian mesh allows for fast, efficient movement of the simulated particles, so that the computational time can be spent in the collision routines, which contain the bulk of the physics. Additional efficiency is obtained with preprocessing CAD routines, which reduce the startup costs of new simulations. These preprocesses allow for easy definition and assembly of geometric primitives to construct simulation geometries sufficient to model even very complex spacecraft.[14,17] The code has been successfully applied to simulate flows over slender hypersonic vehicles,[20,21] blunt re-entry vehicles,[16] and spacecraft (including Magellan[14] and UARS/HALOE[17]). Although the present study examines the ARIES missile at zero angle of attack, the inclusion of the fins creates three-dimensional flow structures. Thus, the use of a three-dimensional approach is necessary. Also, demonstration of three-dimensional capability for these missile flows is desired for future finite angle of attack cases.

The physical models employed in the presented simulations are as follows. Intermolecular collisions are treated using the variable hard sphere model.[4] Energy partitioning is accounted for using the Borgnakke–Larsen phenomenological model[22] with constant rotational and vibrational collision numbers for internal relaxation as suggested by Bird.[9] Conditions in all simulations were such that no finite-rate chemistry occurred. For the gas–surface interactions, fully diffuse reflections with full thermal accommodation were assumed.

The value of a thermal accommodation coefficient has a significant impact on the predicted aerodynamic heating.[5] Since it is not trivial to describe energy accommodation for a given material under a general set of flow conditions, we bypass this issue and assume full thermal accommodation in the DSMC simulations, CFD calculations, free-molecular calculations, and bridging relation derivations. Since no direct comparisons are made with experimental data, the calculations are self-consistent and the issue of thermal accommodation coefficients is irrelevant to discussions contained herein. However, future comparisons of these methods with experimental data will necessitate fully addressing this issue.

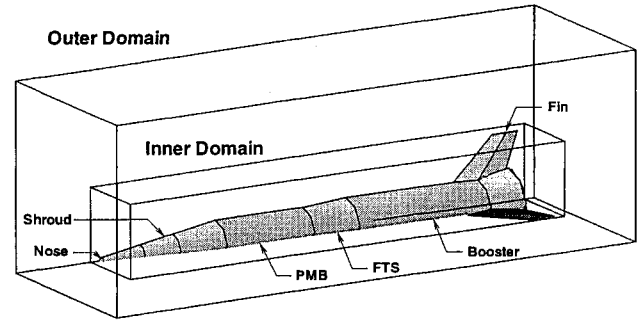


Fig. 1 Definition of two-domain decomposition used for ARIES parallel DSMC simulations.

Recently, Rault et al.[17] extended the coding to allow for parallel processing through use of Oak Ridge National Laboratory's parallel virtual machine software. Thus, multiple workstations and/or multiple CPU workstations can be combined to work on a problem. The simulations conducted in the present study were decomposed into two domains, each run on separate processors of a Sun Sparc-10 or SGI Challenge workstation. As seen in Fig. 1, an inner domain was constructed to barely enclose the missile geometry, and an outer domain surrounds that. Also, the symmetry planes are used to reduce the computational domain to one fourth of the actual missile geometry. In this way, increased resolution is easily obtained near the missile with use of a smaller amount of simulated particles and computational time required. Because the aerodynamic heat transfer is one of the most difficult of the aerodynamic quantities to compute, several simulations were conducted to ensure grid-independent simulations that met all of the DSMC grid requirements.[9] The presented results were for simulation parameters of approximately 100,000 cells for all three cases, with the numbers of simulated particles ranging from 1.5×10^6 for the 138-km case to 2.1×10^6 for the 91-km case. In addition, using the actual surface geometry and employing body layers[11] allowed for accurate resolution of the Knudsen layers next to the surface of the missile to accurately predict the surface properties.

Heat Transfer Bridging Relations

Until recently, the computational time required to obtain three-dimensional particle solutions in the transition regime was excessive for full missile trajectory analyses. Thus, heat transfer bridging relations[6,7] are still widely used. The validity of three bridging relations is examined in this study for the transitional flow over the ARIES missile.

The first heat transfer bridging relation examined is that proposed by Matting.[6] This bridging relation was derived using a simple kinetic theory model and should be applicable to any blunt body. The bridged heat transfer value Q_{br} recommended for supersonic or hypersonic flow is given by[6]

$$Q_{\text{br}} = Q_c[1 - \exp(-Q_{\text{FM}}/Q_c)] \tag{1}$$

This equation is a monotonic function of Q_{FM}/Q_c, and has the correct continuum and free-molecular asymptotes. Note that this relation does not allow for any overshoot above either the continuum or the free-molecular values.

Next, the bridging relation suggested by Nomura[7] was considered. This correlation, which was an extension of a similar correlation for low-speed flows, makes use of the Knudsen number to account for the rarefaction. The bridged heat transfer coefficient $(C_h)_{\text{br}}$ is given by[7]

$$(C_h)_{\text{br}} = \frac{(C_h)_c + (Kn/c)^2 (C_h)_{\text{FM}}}{1 + (Kn/c)^2} \tag{2}$$

where c is a matching constant experimentally determined by Nomura[7] to be 3. The form of the Knudsen number was given as[7]

$$Kn^2 = \frac{V_\infty \mu_\infty}{a_\infty^2 \rho_\infty R_N} = \left(\frac{M_\infty}{\sqrt{Re_\infty}}\right)^2 \tag{3}$$

Note that this form of the Knudsen number is strictly applicable only for large Reynolds numbers.[5] However, it is probably adequate, because the value of c was determined by comparisons with experiment using the Knudsen number defined in this manner. In contrast to the Matting bridging formula given in Eq. (1), the Nomura bridging relation will overshoot either the continuum or the free-molecular value. This bridging relation also has the correct continuum and free-molecular asymptotes.

The third bridging relation examined is a modified version of the Nomura bridging relation. Noting that the length scale in the Knudsen number given by Eq. (3) was only applicable for the spherical nose region of a missile, Murty[18] suggested using the local radius value as the length scale away from the nose region. Thus, the length scale will increase along the conic sections of the ARIES missile. Otherwise the bridging relation is the same as given in Eq. (2).

Free-Molecule Flow

Whether the missile is in the free-molecule flow regime or in the transition flow regime, the ability to calculate the free-molecular flow solution is required. In the transition regime, this is because bridging relations[6,7] need the free-molecular heating values as input.

Kinetic theory offers closed-form analytic expressions for the aerodynamic heating[4]; however, more approximate or limiting forms are widely used.[6,7] Assuming the particle velocities are distributed according to a Maxwellian distribution, the local heat transfer to some surface element with inclination angle θ to the flow is given by[4]

$$\frac{2\beta_\infty^3 Q}{\rho_\infty} = a_c\left(\frac{1-\varepsilon}{2\sqrt{\pi}}\right)\left\{\left[s^2 + \frac{\gamma}{\gamma-1} - \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)\frac{T_w}{T_\infty}\right]\right.$$
$$\times \{\exp[-(s\sin\theta)^2] + \sqrt{\pi}s\sin\theta[1 + \operatorname{erf}(s\sin\theta)]\}$$
$$\left. - \frac{1}{2}\exp[-(s\sin\theta)^2]\right\} \tag{4}$$

The value of γ was assumed to be 1.4, and the speed ratio s and β_∞ are given by[4]

$$s = V_\infty \beta_\infty, \qquad \beta_\infty = (2RT_\infty)^{-\frac{1}{2}} \tag{5}$$

An approximate form of the free-molecular heat transfer may be obtained by assuming $s\sin\theta$ is large and $T_w/T_\infty \approx 1$. Then Eq. (4) reduces to

$$Q = \tfrac{1}{2}\rho_\infty V_\infty^3 \sin\theta \tag{6}$$

Another approximate form widely used comes from a thermodynamic standpoint. Again, assuming full thermal accommodation, the expression for the stagnation point is given by[6,7]

$$Q = \rho_\infty V_\infty (H_\infty - H_w) \tag{7}$$

For surface points away from the stagnation point an additional factor of $\sin\theta$ appears due to geometric considerations. Thus for $T_w \approx T_\infty$, Eq. (7) reduces to Eq. (6).

For free-molecular predictions over the ARIES missile, both approximate forms will predict a heating value of zero when θ vanishes. Therefore, the limiting forms will not be applicable over the cylindrical portions of the missile, namely the payload module bus (PMB) and booster sections.

Results and Discussion

The DSMC method was used to simulate the flow over the ARIES missile in the descent portion of a trajectory corresponding to approximately 138-, 110-, and 91-km altitude. The freestream conditions for each of these trajectory points are presented in Table 1.

Table 1 Freestream flow conditions for the ARIES missile

Altitude, km	138	110	91
V_∞, km/s	2.045	2.178	2.247
ρ_∞, kg/m^3	3.93×10^{-9}	1.24×10^{-7}	2.18×10^{-6}
N_∞, m^{-3}	8.72×10^{16}	2.76×10^{18}	6.20×10^{19}
T_∞, K	677	249	194
$(X_{N_2})_\infty$	0.77	0.77	0.791
$(X_{O_2})_\infty$	0.12	0.12	0.206
$(X_O)_\infty$	0.11	0.11	0.003
T_w, K	300	300	300
α, deg	0	0	0

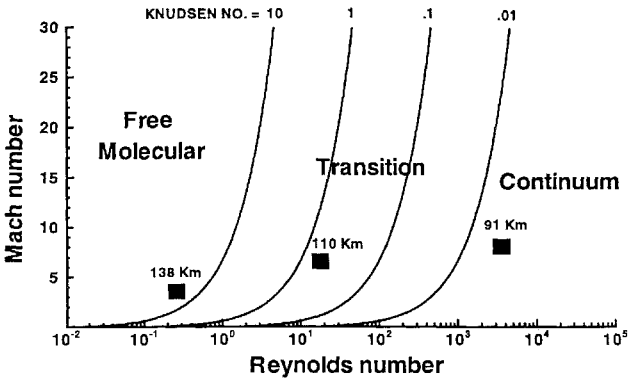


Fig. 2 ARIES trajectory points in Reynolds–Mach–Knudsen number domain.

Using the Knudsen number based on freestream values and missile length, these points correspond to the near free-molecular, transition, and near continuum flow regions, respectively, as shown in Fig. 2. However, because of the length scale based on missile length, these points tend to show Knudsen numbers smaller than realistically expected. Thus, the actual flow characteristics should be somewhat more rarefied at each point. The DSMC method requirements[9] on the computational time step, number of simulated particles per cell, and grid size were met for all the simulations presented in this study.

To calculate the aerodynamic heat transfer from the bridging relations, values are needed from both a continuum and a free-molecular approach. Continuum heat transfer predictions were obtained using an approximate viscous method[2] for the nose and shroud regions, and using other analytical techniques[23,24] for the cylindrical portions and fins. These methods were used for all of the continuum calculations presented in this study. Possible inaccuracies as a result of the approximate nature of these methods were not investigated. The possible impact on later discussions will be indicated.

138-km-Altitude Case

DSMC Simulation

The first trajectory point examined corresponds to an altitude of 138 km. Figure 3 shows the normalized number density N/N_∞ from the DSMC simulation on one radial plane that intersects a fin. As expected, the maximum number density is seen on the nose. Also, large values are noted on the shroud and leading edge of the fin. Further increase, though not as large, is noted on the flight termination system (FTS). Because of the large rarefaction seen in this case, the density increases extend a considerable distance from the surface. Also, no discernible shock is observed.

Free-Molecule Comparison

For the free-molecular heat transfer values, two methods were examined. The first is the approximate method given by Eq. (6). It assumes that the speed ratio times the sine of the local incident angle is large and that the wall to freestream temperature ratio is close to unity. The second method is the full kinetic-theory equations given by Eq. (4) and does not use the above approximations. Figure 4 compares solutions using the two free-molecular approaches along the surface of the ARIES missile for the 138-km-altitude case. The methods show large disagreement along the entire length of the

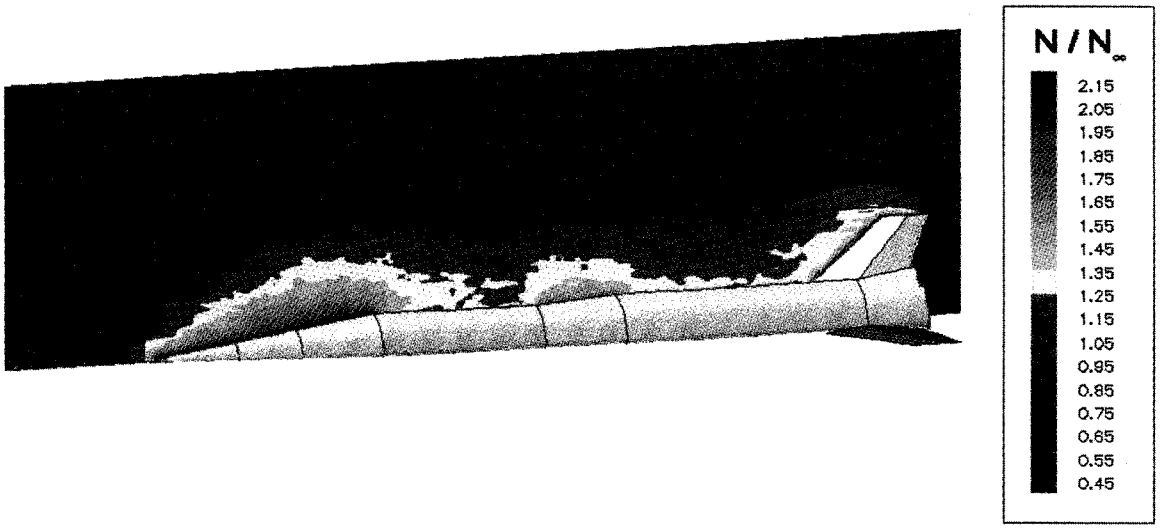


Fig. 3 Normalized number density for 138-km-altitude case.

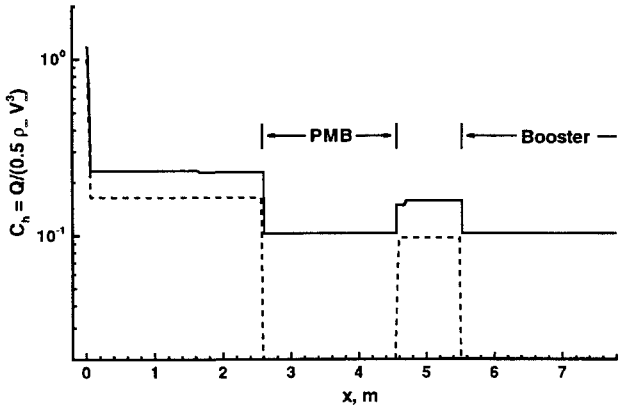


Fig. 4 Comparison of free-molecular heat transfer predictions vs axial distance from nose for 138-km-altitude case: ——, full free molecular expression and - - - - -, approximate free molecular expression.

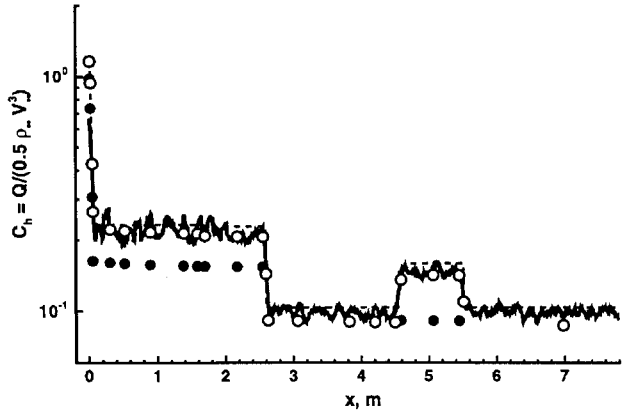


Fig. 5 Effects of free-molecular predictions on bridged heat transfer values using Matting bridging relation for 138-km-altitude case: - - - - -, free molecular; ——, DSMC simulation; •, C_h using Matting with approximate free-molecular expression; and ○, C_h using Matting with full free-molecular expression.

missile. On the cylindrical portions of the missile, i.e., the PMB and booster sections, the local incident angle is zero. Hence, the approximate free-molecular expression predicts no aerodynamic heating over these portions. The discrepancy along the other regions with large incident angles is due to the small speed ratio, 3.18, for this case. Also, the freestream temperature for this case was more than twice the wall value. Thus, the assumptions of a large speed ratio and $T_w \approx T_\infty$ are both invalid. The large freestream temperature is

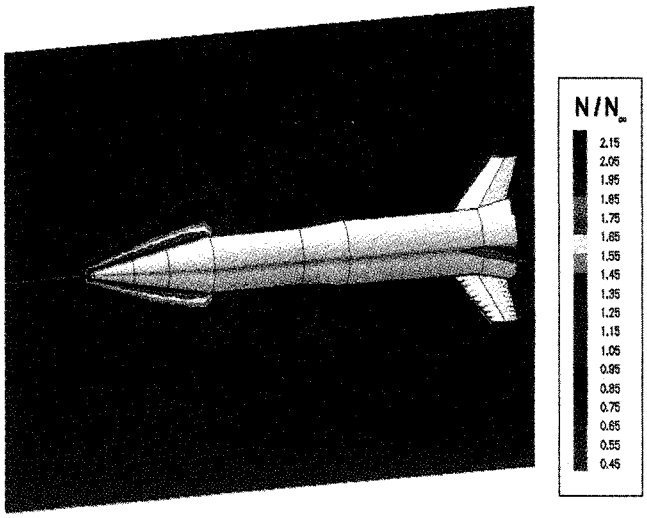


Fig. 6 Normalized number density for 110-km-altitude case.

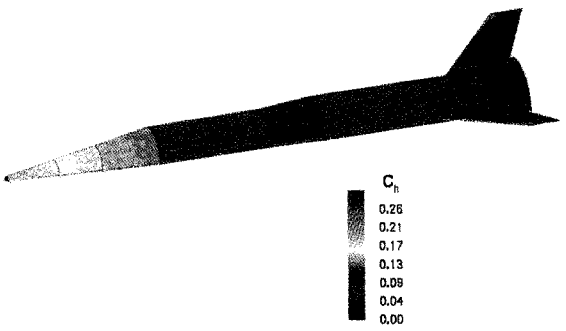


Fig. 7 Surface contour of local heat transfer coefficients for 110-km-altitude case.

due to the high solar heating of the atmosphere that occurs at this altitude. Thus, the full kinetic theory expressions are necessary to predict the free-molecular heat transfer over the missile for this case.

Bridging-Relation Comparison

The effects of using the approximate free-molecular approach for the 138-km-altitude cases are shown in Fig. 5. This figure shows the free-molecular values (from the full kinetic theory expressions), the DSMC heat transfer predictions, and the heat transfer predictions using the Matting bridging relation with the approximate and full free-molecular expressions used as input along the surface of the ARIES missile. As expected, the DSMC method predicts values within a few

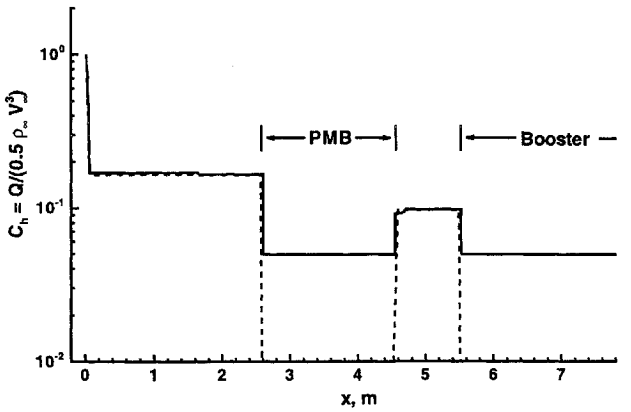


Fig. 8 Comparison of free-molecular heat transfer predictions vs axial distance from the nose for 110-km-altitude case: ——, full free molecular expression and - - - - -, approximate free molecular expression.

percent of the free-molecular values along the length of the missile. Thus, this case is nearly free-molecule flow. The bridging relation predictions using the full free-molecular expressions are in good agreement with DSMC predictions along the length of the missile, differing by less than 5%. However, the bridging relation predictions using the approximate free-molecular expression underpredict the DSMC and free-molecule values on the shroud and FTS. Because the approximate free-molecular expressions predicts zero values along the PMB and booster sections, these points are not included, as they are clearly wrong.

110-km-Altitude Case

DSMC Simulation

In Fig. 6, the normalized number density, N/N_∞, from the DSMC simulation is presented for one plane that intersects the fins. The regions of large density ratio are seen on the nose, shroud, and leading edge of the fin, similar to what was observed for the 138-km case. These three areas all have maximum number densities at least twice the freestream value.

The heat transfer over the surface of the ARIES missile for the 110-km-altitude case predicted by the DSMC simulation is shown in Fig. 7. As expected, the largest heat transfer values are clearly seen on the nose and beginning of the shroud. As will be seen in later figures, the maximum heat transfer value is located at the stagnation point, being nearly unity. The lowest heat transfer values (excluding the base region) are on the cylindrical portions of the missile, namely the PMB and booster sections. Unlike the nose, which is heated primarily by pressure forces, the fins are heated primarily by shear forces. As can be seen, there is significant heat transfer near the leading edges of the fins, indicating relatively large shear forces there. Thus, the heat transfer values over the surface of the ARIES missile predicted by the DSMC simulation follow expected trends.

Free-Molecule Comparison

For the 110-km-altitude case, the approximate and full kinetic-theory free-molecular predictions are shown in Fig. 8 along the surface of the ARIES missile. The speed ratio for this case was nearly 5.8, and the freestream and wall temperatures were much closer than was observed for the 138-km-altitude case. Thus, the approximate free-molecular expression would be expected to agree more closely with the full kinetic theory expression than in the previous case. Although both methods predict nearly the same values over much of the missile length, they differ significantly on the cylindrical portions of the missile where the local angle of incidence vanishes. Thus, the full kinetic theory expressions are still necessary to correctly predict free-molecule heat transfer along the PMB and booster sections.

Bridging-Relation Comparison

Next, the heat transfer coefficients obtained from three bridging relations using the free-molecule values from the full kinetic theory expressions were calculated. Figure 9 compares these values with

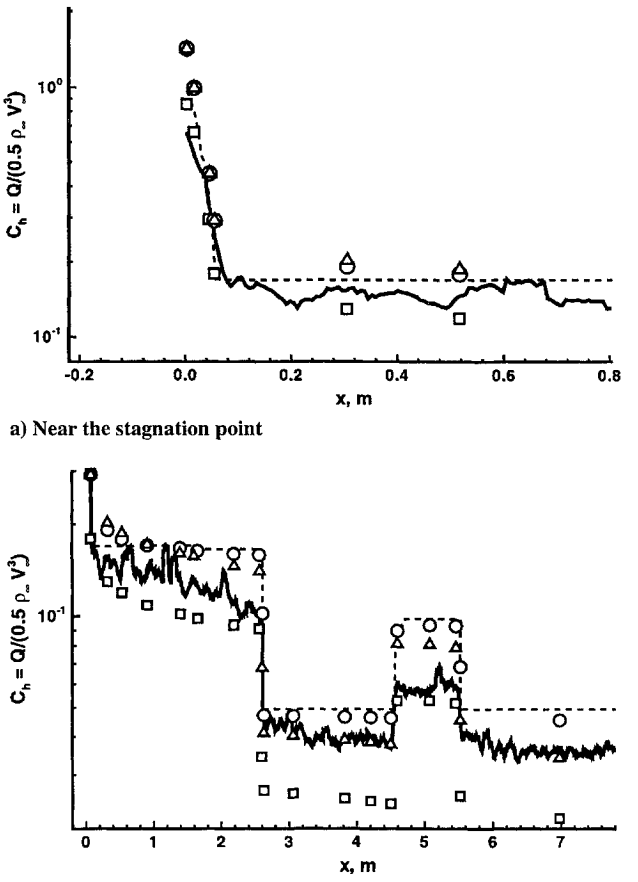


Fig. 9 Comparison of heat transfer predictions between different bridging relations and DSMC simulation for 110-km-altitude case: - - - - -, free molecular; ——, DSMC simulation; □, C_h using Matting bridge relation; ○, C_h using Nomura bridge relation; and △, C_h using modified Nomura bridge relation.

both the DSMC simulation and the free-molecular values along the surface of the missile for the 110-km-altitude case.

Near the stagnation point, seen in Fig. 9a, the Matting bridging relation predicts values much closer to the DSMC simulation values than the Nomura and modified Nomura bridging relations. The Matting results overpredict the DSMC values slightly at the stagnation point, whereas the Nomura and modified Nomura overpredict the DSMC values by more than a factor of 2. Also, the Nomura and modified Nomura predict values nearly 50% above the free-molecular value. Clearly, the heat transfer values predicted near the stagnation point should fall below the free-molecular value.

On the shroud, seen in Fig. 9b, the Matting relation slightly underpredicts the DSMC values by nearly the same amount that both the Nomura and modified Nomura methods overpredict. Considering the previous statements about the possible inaccuracies of the approximate continuum values, a lower continuum value would not change the Matting values in this region by much. However, the Nomura and modified Nomura values could be reduced and lie much closer to the DSMC values. Since the modified Nomura bridging relation reduces to the Nomura bridging relation on the nose, both give the same values there. On the PMB, the Matting bridging relation underpredicts values compared to the DSMC results, while the Nomura bridging relation slightly overpredicts them. The modified Nomura predicts values closest to the DSMC results in this region, differing by less than a few percent along the length of the PMB. On the FTS, the opposite trend is seen. The Matting predictions now lie within a few percent of the DSMC values along the length of the FTS, while both the Nomura and modified Nomura overpredict in this region. Then, along the booster section, the agreement is similar to that viewed on the PMB.

Since the DSMC predicted values lie close to the free-molecular values along the length of the missile, this suggests that this case is

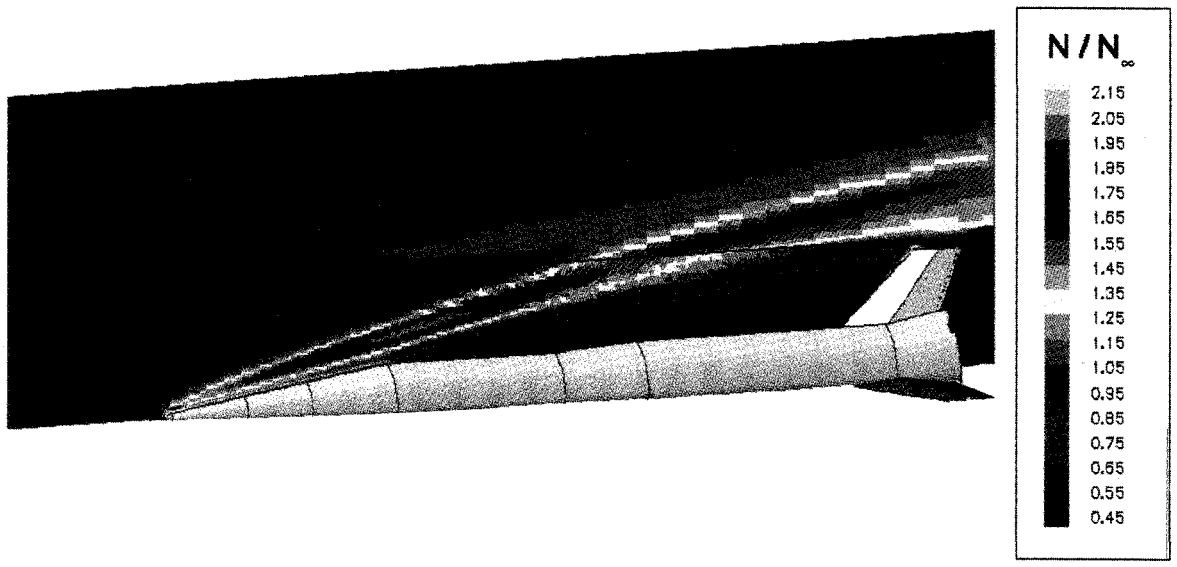


Fig. 10 Normalized number density for 91-km-altitude case.

closer to free-molecule flow than to continuum flow. Although the freestream Knudsen number based on the missile length suggests that it lies nearly midway between free-molecule and continuum flow, local values based on local body dimensions range from approximately 6.8 on the nose to 2.1 near the tail. Thus, local Knudsen number values further support that this case lies closer to the free-molecule end of the transition regime.

91-km-Altitude Case

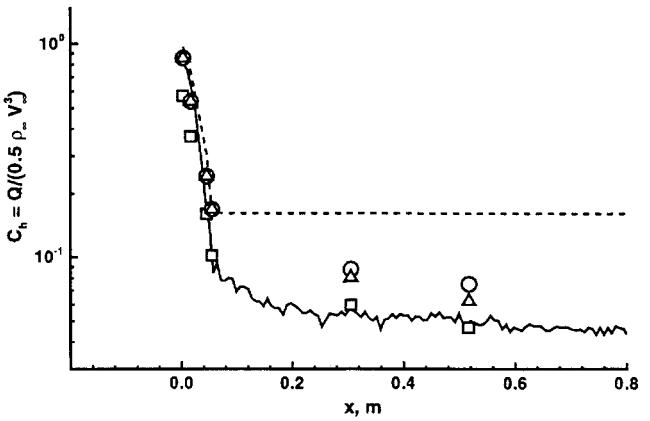
The last trajectory point examined corresponds to an altitude of 91 km. Because of the larger local Knudsen numbers obtained for the 110-km case, this trajectory point deeper in the transition regime was examined to compare surface heating predictions from DSMC simulations and bridging relation approaches. Since the previous two cases showed the necessity of employing the full free-molecular expression given by Eq. (4) with the various bridging-relations, all bridging relation predictions presented for this case will use this expression.

DSMC Simulation

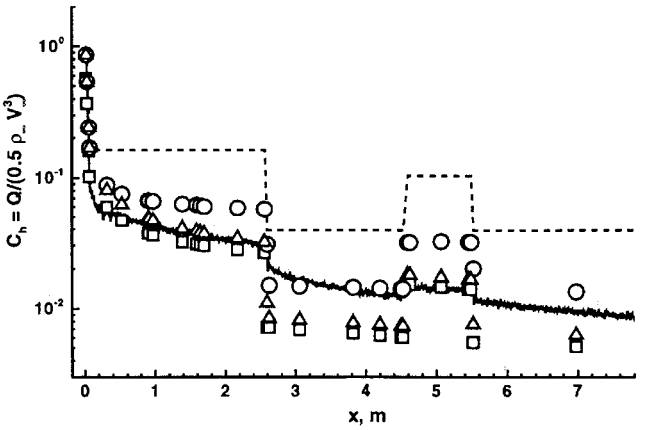
Figure 10 shows the normalized number density N/N_∞ from the DSMC simulation on one radial plane that intersects a fin. A clear conical shock formation is observed, indicating a less rarefied flowfield than previously observed in the other two cases. Figure 11 compares heat transfer coefficients along the surface of the missile from the three bridging relations with both the DSMC simulation and the free-molecular results. The DSMC predictions fall much farther below the free-molecular values for the 91-km case than was seen (Fig. 9) for the 110-km case. This drop in heating from the free-molecular values shows the increased effect of intermolecular collisions on heat transfer and suggests a more transitional flowfield than observed for the 110-km case.

Bridging Relation Comparison

At the stagnation point, shown in more detail in Fig. 11a, the Nomura and modified Nomura bridging relation predictions are much closer to the DSMC results than the Matting bridging relation predictions. By the beginning of the shroud, though, the Matting values are in much better agreement, and the Nomura and modified Nomura values exceed the DSMC predictions. Along the rest of the missile length, shown in Fig. 11b, the following trends are noted. On the shroud and FTS, the modified Nomura and Matting predictions are in good agreement, while the Nomura predictions differ by more than a factor of two. On the PMB, the Nomura predictions are in close agreement, but the modified Nomura and Matting values underpredict the DSMC results. On the booster section of the missile, all three bridging relation predictions are in poor agreement with the DSMC predictions. Thus, following the same trend seen in the 110-km case, no one bridging relation performed best along the entire length of the missile.



a) Near the stagnation point



b) Along the missile length

Fig. 11 Comparison of heat transfer predictions between different bridging relations and DSMC simulation for 91-km-altitude case: - - - - -, free molecular; ——, DSMC simulation; □, C_h using Matting bridging relation; ○, C_h using Nomura bridging relation; and △, C_h using modified Nomura bridging relation.

Examining the local Knudsen numbers given by the modified Nomura relation along the length of the missile gave values from 1.66 at the nose to approximately 0.5 near the tail. These Knudsen numbers suggest that this case is near the middle of the transition regime. Since one would expect the bridging relations to perform

worse in the middle of the transition regime, the larger discrepancies noted for this case are not surprising.

Concluding Remarks

The three-dimensional flow over the ARIES missile was simulated using the DSMC method for three trajectory points corresponding to transition and near free-molecule flow. The transition regime points were used to assess the performance of three empirical bridging relations over a slender body, namely the bridging relation from Matting, the bridging relation from Nomura, and a modified form of the bridging relation from Nomura. Also, the effects of using an approximate free-molecular expression were investigated.

Comparisons of the free-molecular solutions at the 138- and 110-km trajectory points showed that the full free-molecular expression given by kinetic theory was necessary to predict the aerodynamic heating along the entire length of the ARIES missile. Although the approximate free-molecule expression performed well over the nose and conical regions of the missile for freestream temperatures close to the wall temperature and larger speed ratios, it failed to predict any heating over the cylindrical portions of the body. For conditions when the freestream and wall temperatures differed and the speed ratios were smaller, the approximate free-molecular approach differed significantly along the entire length of the missile.

Using the full free-molecular expression from kinetic theory, the three bridging relations predicted heating rates within a factor of 2 of the DSMC simulation results. Generally, the Matting relation predicted heating rates that compared well with the DSMC results along the nose and conical regions of the missile, but underpredicted values along the cylindrical portions. Conversely, the Nomura and modified Nomura relations generally overpredicted heating rates along the nose and conical regions of the missile, but were much closer along the cylindrical portions. Furthermore, the modified Nomura usually gave better agreement than the Nomura relation with the exception of the 91-km case along the PMB. However, no one bridging relation was seen to outperform the others or agree well with the DSMC simulation along the entire length of the missile.

In conclusion, the findings of this study show that bridging relations for transition regime aerodynamic heating of missiles differ significantly from DSMC predictions. The importance of accurate free-molecular input has been established. The bridging relation approaches were expected to perform well for these benign cases with 0-deg angle of attack and no chemistry, yet large discrepancies were observed. Future work should examine angle of attack effects and chemistry effects that will result from higher missile velocities.

Furthermore, the state of three-dimensional DSMC simulations has progressed considerably. With the quick setup times using CAD preprocessors, and efficient coding including parallel algorithms for fast runs, the DSMC method is capable of providing the necessary accurate heat transfer data for these missile configurations in a relatively short time. Since none of the bridging relations examined were accurate along the entire missile length, this option should be considered for future trajectory analyses.

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